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Shapley Value

Used in horizontal logistics collaborations

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Gain sharing in a horizontal collaboration among shippers: implementing the Shapley value

When a consortium of shippers has been established, one of the trustee's tasks is to calculate the cost reductions for each of the shippers involved. Since we use cooperative game theory in this step, we first recall some basic notions from game theory. Myerson (1991) defined game theory as "the study of mathematical models of conflict and cooperation between intelligent and rational decision-makers. Game theory provides general mathematical techniques for analyzing situations in which two or more individuals make decisions that will influence one another's welfare". Cooperative game theory focuses on cooperative behavior by analyzing and simulating the negotiation process within a group of shippers in establishing a contract or joint plan of activities, including an allocation of collaboratively generated revenues or collaboratively avoided cost. In particular, the possible levels of cooperation and the revenues of each possible coalition (a subgroup of the shippers' consortium) are taken into account so as to allow for a better comparison of each shipper's role and impact within the group as a whole. In this way, shippers in a coalition can settle on a compromise allocation in an objectively justifiable way, moderated by the trustee. Having this in mind, the game underlying horizontal cooperation among a consortium of shippers is evidently a cooperative game. The problem of allocating the jointly generated synergy savings is critical to any logistics cooperation (cf. Thun (2003)).

Let N be a finite set of shippers and denote by 2^N the collection of all subsets of N. Elements of 2^N are called *coalitions, N* is referred to as the *grand coalition*. The cost savings that a coalition S can jointly generate without the shippers in $N \setminus S$ is called the *value* of coalition S. The values of all coalitions S are captured in the so-called characteristic function $v: 2^N \rightarrow \mathbb{R}$. The Shapley value (Shapley (1953)) is a well-known solution concept that constructs a vector $\Phi(N,v) \in \mathbb{R}^N$ that allocates the value $v(N)$ of the grand coalition based on the values $v(S)$ of all coalitions S. The Shapley value can be explained as follows. Consider the creation of a coalition S to which i does not belong. First, a set size $|S|$ is chosen at random out of $\{0,1,2,...,|N|-1\}$, each having a probability $\frac{1}{|N|}$ to be drawn. Then a subset of $N \setminus \{i\}$ of size $|S|$ is chosen, each with a probability $\frac{|S|!(|N|-1-|S|)!}{(|N|-1)!}$. After S has been drawn, shipper i is allocated his so-called *marginal contribution* $v(S \cup \{i\}) - v(S)$. Then, the Shapley value is the expected payoff for shipper i in this random procedure, as indicated in formula (1):

$$\Phi_i(N,v) = \sum_{S \subset N: i \notin S} \frac{|S|!(|N|-1-|S|)!}{|N|!}\left[v(S \cup \{i\}) - v(S)\right], \text{ for all } i \in N. \qquad (1)$$

The Shapley value possesses a number of objective fairness properties. Below we will briefly discuss four of these properties that are useful in the context of horizontal cooperation between a consortium of shippers. First, the *efficiency* property of the Shapley value ensures that the total value of the grand coalition is distributed among the shippers, i.e., no value is lost. The Shapley is also *symmetric*, meaning that two shippers that create the same additional value to any coalition receive the same share of the total value. The *dummy* property states that shippers that do not contribute anything to any coalition except their individual value indeed receive exactly their individual value as a final share of the total value. Finally, we mention the Shapley value's property of *strong monotonicity.* This guarantees that if all of the shipper's marginal contributions increase, his payoff will increase. Since these four properties make perfect sense from a practical perspective, we make use of the Shapley value as our gain sharing methodology.

In order to cover the overhead costs needed to service the shippers and to moderate the consortium efficiently to gain profit, the trustee claims a pre-determined percentage of the savings attained as a result of synergy. This percent claim is called the *synergy claim* and is denoted by $p \in [0,1]$. The consortium should decide on the level of the synergy claim, depending on the activities to be undertaken by the trustee.

The value $v(S)$ of a coalition S in the gain sharing game is determined by means of formula (2):

$$v(S) = (1-p)\max\left\{\sum_{i\in S} C_0(i) - C(S), 0\right\} \qquad (2)$$

Here, $C_0(i)$ are the costs of shipper i in the status quo situation, i.e., when shipper i privately performs his transportation orders, while $C(S)$ represents the costs of the LSP to collectively execute the orders $\bigcup_{i\in S} O_i$ of all shippers in S. Obviously, a coalition S can only be established when this consortium can perform at a lower cost level than the sum of the costs that the shippers in S incur when they would all perform their own orders individually. Whenever this is not the case, the shippers in S will stay in the consortium, and this coalition is left out of consideration. $v(S)$ is then set to 0. This explains the use of the maximum with 0 in (2).

References

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